TeV scale horizontal gauge symmetry and its implications in B-physics

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Abstract

We propose a gauged $U(1)_H$ horizontal symmetry around TeV scale that is a subgroup of a $SU(3)_H$ horizontal gauge symmetry broken at $\mathcal{O}(10^{14})$ GeV. The breaking generates right-handed Majorana neutrino masses through a $SU(3)_H$ sextet scalar. A particular Majorana right-handed neutrino mass matrix explicitly determines the remnant $U(1)_H$ at low energy which only couples to $b - s$ and $\mu - \tau$ in the gauge eigenstate. The dangerous $K - \bar{K}$, $D - \bar{D}$ mixing and $B_s \to \mu^+ \mu^-$ are kept to be safe because the relevant couplings are suppressed through high powers of small mixing angles in the fermion rotation matrix. Our analysis which applies to the general case shows that the Tevatron di-muon anomaly can be explained through the B_s and B_d mixing while keeping all the other experimental constraints within 90 % C. L. For the B meson decay, the $B_s \to \mu^\pm \tau^\mp$ is the leading leptonic decay channel which is several orders of magnitude below current experimental bound.

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I. INTRODUCTION

Horizontal gauge symmetry was proposed as an extension of the SM gauge symmetries to unify all families of quarks and leptons [1, 2]. Given the three families of quarks and leptons, $SU(3)_H$ is the most natural choice for the horizontal gauge symmetry. Interestingly, if one assumes all the SM fermions transform under $\mathbf{3}$ of $SU(3)_H$, the anomaly free condition requires three generations of right-handed neutrinos $n_R^{i=1,2,3}$ [3] while the right-handed neutrinos also play important roles in explaining the origin of neutrino masses. Therefore, the $SU(3)_H$ horizontal gauge symmetry model provides a natural scheme for the seesaw mechanism [4] generating small masses for light neutrinos [3]. The Majorana neutrino mass term for the right-handed neutrinos explicitly breaks the $SU(3)_H$, thus it is often believed that the horizontal gauge symmetry should be broken at a very high-energy scale $M_R \sim \mathcal{O}(10^{14})$ GeV. Then it seems impossible to test the $SU(3)_H$ gauge interactions in low-energy experiments. However, it is not always the case as we will show in detail below. Even if some subgroup of the $SU(3)_H$ remains unbroken, the right-handed neutrinos can still acquire large Majorana masses.

Since n_R^i transform as $\mathbf{3}$ under $SU(3)_H$, the Majorana neutrino mass term can arise from the vacuum expectation value (*vev*) of an $SU(3)_H$ sextet χ_6,

$$\overline{n_R^{ic}}\langle\chi_6\rangle_{ij}n_R^j, \tag{1}$$

and $M_R = \langle\chi_6\rangle$. The light neutrino mass is given by the seesaw mechanism as $m_\nu = m_D^T(M_R)^{-1}m_D$. In order to explain the the neutrino oscillation data, suitable $\langle\chi_6\rangle$ and m_D are required. For m_D and the other SM fermion masses, there must exist octet Higgs under $SU(3)_H$ in order to accommodate the correct mass hierarchy in quarks and leptons. In addition, to minimize flavor changing effects induced by the octet Higgses, we employ a scenario with additional Higgses and singlet fermions [5] in which the m_D and quark mass matrices or lepton mass matrix are all independent. By taking a suitable gauge choice of horizontal symmetry, we always choose the M_R to be a diagonal matrix,

$$M_R = \langle\chi_6\rangle = \begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{pmatrix} \tag{2}$$

The $\langle\chi_6\rangle$ structure explicitly determines the symmetry breaking. The $SU(3)_H$ is completely

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broken in a generic vacuum with $A \neq B \neq C(\neq A)$. However, with a specific vacuum of $A = B = C$ for instance, the vacuum $\langle \chi_6 \rangle$ is invariant under a $SO(3)$ symmetry and the breaking is $SU(3)_H \to SO(3)_H$. Being symmetric second rank tensor under $SU(3)_H$, the sextet χ_6 transform as $\chi_6 \to U^T \chi_6 U$ where $U = e^{i\epsilon_a T_a}$ and T_a is the generator of the horizontal symmetry. A general scheme to obtain the unbroken symmetry is derived from the condition that if $\{T, \langle \chi_6 \rangle\} = 0$, $\langle \chi_6 \rangle$ is invariant under transformation defined by T. To illustrate the feature of our proposal, we take a vacuum as $C = -B$. This vacuum $\langle \chi_6 \rangle = \text{diag}(A, B, -B)$ is invariant under the $SU(3)$ generator λ_6 as

$$
T = \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} . \tag{3}
$$

Consequently, one can identify the unbroken $U(1)_H$ gauge symmetry with generator T [1] and it can survive to low energy, for instance $\mathcal{O}(\text{TeV})$ which may lead to interesting predictions in flavor changing neutral current (FCNC) processes. It was also observed that if there exist horizontal gauge interactions, CP violation can be realized with only two generations. Explicit examples of CP violation due to $U(1)_H$ and $SU(2)_H$ was discussed in [1]. If the above $U(1)_H$ is broken at the low energy, the horizontal gauge boson exchanges can induce additional CP violations [1] at low energies through quark and lepton mixings.

In the last decades, huge experimental efforts had been made in improving the measurements on CP violation in the B meson system. Very recently, the DØ Collaboration at Tevatron has reported a large charge asymmetry in like-sign di-muon $A_{s\ell}^b$ in both B_s and B_d decays with 6.1 fb^{-1}.

$$
A_{sl}^b(\text{Exp}) \equiv \frac{N^{++} - N^{--}}{N^{++} - N^{--}} = -9.57 \pm 2.51(\text{stat.}) \pm 1.46(\text{syst.}) \times 10^{-3} , \tag{5}
$$

[1] This $U(1)$ T has an unitary equivalent representation. By taking a $45°$ rotation R between 2nd and 3rd axes in horizontal space, the *vev* becomes

$$
R^T \langle \chi_6 \rangle R = \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & B \\ 0 & B & 0 \end{pmatrix} . \tag{4}
$$

The $U(1)$ then becomes $T' = R^\dagger T R = \text{diag}(0, 1, -1)$. We take, throughout this paper, the basis where the Majorana mass matrix is diagonal as $M_R = \text{diag}(A, B, -B)$ and the unbroken $U(1)_H$ generator is given by T in Eq. (3).

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where $N^{++}(N^{--})$ is the event number for $b\bar{b} \to \mu^+\mu^+X(\mu^-\mu^-X)$. Such a large di-muon charge asymmetry has a $3.2\,\sigma$ deviation from the SM prediction $A_{sl}^b(\text{SM}) = (-2.3^{+0.5}_{-0.6}) \times 10^{-4}$ [8] and many models have been proposed to account for this anomaly [17–20]. The CDF has also measured $A_{sl}^b = 8.0 \pm 9.0 \pm 6.8 \times 10^{-3}$ [21], using 1.6 fb^{-1} of data, which has a positive value and large uncertainties. Combining the above two results in quadrature (include the systematic uncertainty), we have

$$A_{sl}^b \simeq -8.5 \pm 2.8 \times 10^{-3} \ . \tag{6}$$

At the Tevatron both B_d and B_s mesons are produced, hence A_{sl}^b is related to the charge asymmetries $a_{sl}^{d,s}$ in B_d and B_s decays by [2]

$$A_{sl}^b = (0.506 \pm 0.043)a_{sl}^d + (0.494 \pm 0.043)a_{sl}^s \ . \tag{7}$$

New physics (NP) contributions in B_d mixing are strictly constrained (we will show it more explicitly in the parameter fit later), so only large NP contributions to the B_s mixing (comparing to the other meson mixings) are allowed. For the NP contribution, if the mixing in the rotation matrix between mass eigenstate and gauge eigenstate is not huge, then one would naturally expect the $U(1)_H$ that maximizes $b-s$ mixing as in Eq. (3). Indeed, for a CKM-like rotation matrix, the gauge boson coupling matrix at the tree level in the mass basis goes like

$$G \sim \begin{pmatrix} \lambda^4 & \lambda^3 & -\lambda \\ \lambda^3 & -\lambda^2 & 1 \\ -\lambda & 1 & \lambda^2 \end{pmatrix} , \tag{8}$$

where λ is the Wolfenstein parameter [22] around the order of the Cabibbo angle ($\lambda \simeq 0.1$). Clearly, the meson mixings between the first two generation are highly suppressed. The NP also couples to leptons. However, their contributions to the B meson decay branching ratio to electron and muon are highly suppressed (although λ should be replaced by some small

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[2] If the semileptonic b-hadron decays do not involve CP violating phase, then the charge asymmetry is directly related to the mixing-induced CP asymmetris in B_d and B_s meson oscillations. $a_{\text{fs}}^s = -(1.7 \pm 9.1 \pm 1.5) \times 10^{-3}$ [11] is measured through the time dependence of $B_s^0 \to \mu^+ D_s^- X$ and its CP conjugate at D0 and $a_{\text{SL}}^d = -(4.7 \pm 4.6) \times 10^{-3}$ [10] at the B factory. Nevertheless, we do not use those results here due to their large uncertainty.

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mixing of the lepton rotation matrix) [3]. Therefore, we focus on the phenomenology in the B meson mixing and decay.

The paper is organized as follows: in section II, we propose the specific model in which we consider in the paper. In Section III we show phenomenological implications of our model on flavor physics which has subsection III A related to meson mixing and subsection III B related to meson decay. Section IV contains our conclusions.

II. THE MODEL

The model starts with a gauged $SU(3)_H$ model at extremely high energy. By taking all the fermions as **3** under $SU(3)_H$. The particle contents under $SU(3)_H \times SU(2)_L \times U(1)_Y$ is

$$q_L : (3, 2, \frac{1}{3}), \quad u_R : (3, 1, \frac{4}{3}), \quad d_R : (3, 1, -\frac{2}{3})$$
$$\ell_L : (3, 2, -1), \quad e_R : (3, 1, -2), \quad n_R : (3, 1, 0) \tag{10}$$

which is exactly vectorial and the $SU(3)_H$ is therefore anomaly free symmetry. It is crucial to have right-handed neutrino triplets, $n_R^{i=1,2,3}$, for anomaly cancellation [3]. The extension to the Pati-Salam unification [23] may be straightforward.

As we have already discussed the sextet breaking in the introduction, here we focus on the Yukawa interactions for the other SM fermions and the Dirac neutrino mass matrix. In conventional $SU(3)_H$ models, in order to break the $SU(3)_H$ as well as the $SU(2)_L \times U(1)$, one usually introduces one $H : (1, 2, 1)$, four $\Phi_8 : (8, 2, 1)$ to generate all the SM fermion mass hierarchies [4]. However, the $(8, 2, 1)$ Higgs will induce large FCNC [24] if the Higgs is light. To avoid the too large FCNC problem, another proposal is to introduce $(8, 1, 0)$

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[3] Comparing to the other NP coupling matrix in the gauge eigenstate G = diag (1, 1, a) [26], which has a coupling matrix

$$G \sim \begin{pmatrix} 1 & a\lambda^5 & (a-2)\lambda^3 \\ a\lambda^5 & 1 & (a-1)\lambda^2 \\ (a-2)\lambda^3 & (a-1)\lambda^2 & a \end{pmatrix} \tag{9}$$

in the mass eigenstate, the lepton decay branching ratio, especially the one to muon is much more suppressed in our case.

[4] Another possibility is to consider bulk $SU(3)$ horizontal gauge symmetry broken at one boundary brane where all fermion masses are generated at that brane. In this case, one do not need color octet Higgses to generate SM fermion hierarchies. See Ref. [27] as an example.

Higgs[5].

$$\Phi_8^i : (8, 1, 0), H : (1, 2, 1) \quad (i = u, d, e, \nu) \tag{11}$$

In addition, to generate effective Yukawa couplings, a new set of $SU(2)_L$ singlet fermions is introduced

$$U_L : (3, 1, \frac{4}{3}), \quad D_L : (3, 1, -\frac{2}{3}), E_L : (3, 1, -2), N_L : (3, 1, 0)$$
$$U_R : (3, 1, \frac{4}{3}), \quad D_R : (3, 1, -\frac{2}{3}), E_R : (3, 1, -2), N_R : (3, 1, 0) \ . \tag{12}$$

These singlet fermions form invariant Dirac masses and act as messengers to generate the necessary Yukawa interactions. We take the up-type quark mass matrix as an example. Since the octet Higgs is no longer $SU(2)_L$ doublet, $q_L \bar{u}_R \Phi$ is forbidden and the up-quark Yukawa interactions only arise as

$$U_L \Phi_8^{(u)} \bar{u}_R + M^U \bar{U}_L U_R + q_L \bar{U}_R H + \lambda_u q_L \bar{u}_R H \tag{13}$$

where $q_L \bar{u}_R H$ is universal. After integrating out the heavy fermion fields U_L, U_R, the effective up-quark Yukawa coupling reduce to

$$\bar{u}_R^i (\lambda_u \delta_{ij} + (\langle \Phi_8^{(u)} \rangle M_U^{-1})_{ij}) q_L^j H. \tag{14}$$

The same mechanism also applies to the mass generation of down type quarks, charged leptons as well as Dirac neutrinos. By assigning the $\langle \Phi_8^{(i)} \rangle$ independently, the mixings and masses in different fermion sectors are completely independent for each other and one can easily accommodate hierarchies and mixings in SM fermions and the Dirac neutrinos. This also enables us to choose the Dirac neutrino mass matrix other than nearly-diagonal structure.

After electroweak symmetry breaking, the effective Yukawa coupling of $\Phi_8^{(u)}$ also arises as

$$\langle H \rangle M_U^{-1} \bar{u}_R \Phi_8^{(u)} u_L \ . \tag{15}$$

Then, the $\Phi_8^{(i)}$ exchanges induce FCNC's in general. We have checked that they satisfy the strongest constraint from K-K mixing, marginally [5]. However, actual effects depends on the mass spectrum of the $\Phi_8^{(i)}$ and hence we do not discuss them in this paper.

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[5] Since $M \simeq \langle \Phi_8 \rangle = M_{Z'}/g_H \simeq 50$ TeV, the effective coupling here is $\langle H \rangle / M$ of $\mathcal{O}(10^{-3})$. With additional propagator suppression due to $1/M^2$, the amplitude is $\mathcal{O}(10^{-15})$ GeV^{-2}.

Another consequence is that both up and down quark mass matrices become Hermitian

$$m_u^\dagger = m_u, m_d^\dagger = m_d \tag{16}$$

Thus, the CP violation in strong interactions due to quark mass matrices, $\arg\{\det(m_u)\det(m_d)\}$ is absent at least at the tree level[5]. In addition, the Hermit mass matrices also require the rotations U_L, U_R in the mass diagonalization $U_L^\dagger m_u U_R$ to be equal $U_L = U_R$. In this case, the horizontal gauge boson couples to vector currents of quarks and leptons. As a consequence, pseudo-scalar bosons like B_s or B_d do not decay to a pair of leptons. However, this is only the result of our specific choice of mass generation model for quarks and leptons. In the following analyses, we assume more generic rotation matrices and U_R and U_L are taken independent for each other to estimate the prediction.

III. PHENOMENOLOGICAL IMPLICATIONS IN FLAVOR PHYSICS

The horizontal gauge interaction is real but family dependent. After the mass diagonalization, the other flavor violation entries as well as new CP violation can arise. The Lagrangian of gauge interactions is

$$
\begin{aligned}
-\mathcal{L}_H &= g_H \bar{q}'_L T \gamma^\mu q'_L Z'_\mu + L \leftrightarrow R \\
&= g_H \bar{q}_L^i \left(V_L^{q\dagger} T V_L^q \right)_{ij} \gamma^\mu q_L^j Z'_\mu + L \leftrightarrow R ,
\end{aligned}
\tag{17}
$$

where V_L^q stands for the rotation for left-handed q-type quarks and T is the generator of $U(1)_H$ interaction given in Eq. (3).

Flavor changing interactions in the SM can only be measured via electroweak charged current interactions. Therefore, for the SM fermion rotation matrixes, only the left-handed ones get constrained from the CKM matrix $V_L^u (V_L^d)^\dagger = V_{CKM}$ and one cannot determine even V_L^u and V_L^d respectively. The other rotations are completely unknown. For simplicity of the discussion here, we will assume that all magnitudes of the left-handed mixings are CKM-like but the complex phases are $\mathcal{O}(1)$ and unconstrained right-handed mixings have the similar structure. Therefore, we have the mixing matrix in the mass eigenstates as

$$(G')_{L/R}^{u/d} = (V_{L/R}^{u/d})^\dagger T (V_{L/R}^{u/d}) \to G' \sim \begin{pmatrix} \lambda^4 & \lambda^3 & -\lambda \\ \lambda^3 & -\lambda^2 & 1 \\ -\lambda & 1 & \lambda^2 \end{pmatrix} \tag{18}$$

This $U(1)_H$ gauge interaction maximizes the mixing in between second and third generations. Mixing magnitude in B_s, B_d and K^0 is at the order of $(1 : \lambda^2 : \lambda^6)$. The $D^0 - \bar{D}^0$ mixing is also at the order of λ^6 suppression comparing with B_s mixing. This $U(1)_H$ is consistent with the phenomenological constraints among different meson mixings. If one assume the lepton doublet and right-handed singlet rotations are the similar to the quark sector [6], one can also compute the leptonic decay of mesons. For instance, $B_s \rightarrow \mu^+ \mu^-$ decay partial width has a λ^4 suppression.

A. Meson Mixing

At the energy scale m_b, the effective Hamiltonian responsible for neutral meson mixing (and in particular $B_s - \bar{B}_s$ mixing) through the tree-level exchange of Z' is

$$\mathcal{H} = C_{LL}^{ij}(m_b)O_{LL}^{ij} + C_{RR}^{ij}(m_b)O_{RR}^{ij} + C_{LR}^{ij}(m_b)O_{LR}^{ij} + \widetilde{C}_{LR}^{ij}(m_b)\widetilde{O}_{LR}^{ij}, \qquad (19)$$

where the $\Delta F = 2$ operators are given by

$$O_{LL}^{ij} = \bar{q}_i\gamma^\mu P_L q_j \bar{q}_i \gamma_\mu P_L q_j, \quad O_{RR}^{ij} = \bar{q}_i\gamma^\mu P_R q_j \bar{q}_i \gamma_\mu P_R q_j,$$
$$O_{LR}^{ij} = \bar{q}_i\gamma^\mu P_L q_j \bar{q}_i \gamma_\mu P_R q_j, \quad \widetilde{O}_{LR}^{ij} = \bar{q}_i P_L q_j \bar{q}_i P_R q_j. \,, \qquad (20)$$

and the Wilson coefficients at $M_{Z'}$ scale are $(\widetilde{C}_{LR}^{ij}(M_{Z'}) = 0)$

$$C_{LL}^{ij}(M_{Z'}) = \frac{g_H^2}{M_{Z'}^2}(G_L'^{ij})^2 \quad C_{RR}^{ij}(M_{Z'}) = \frac{g_H^2}{M_{Z'}^2}(G_R'^{ij})^2 \quad C_{LL}^{ij}(M_{Z'}) = \frac{g_H^2}{M_{Z'}^2}(G_L'^{ij})^2 \qquad (21)$$

where g_H is the horizontal gauge coupling at $M_{Z'}$ scale and $M_{Z'}$ is the horizontal gauge boson mass.

In order to calculate the B physics observables, one has to take into account the running effects of the four operators above. The relation between these four operators at the $M_{Z'}$ and m_b scale is presented in Appendix A. After one obtains the Wilson coefficients at m_b

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[6] Within the minimal $SU(5)$ or $SO(10)$ grand unification theory (GUT), both left-handed and the conjugate of right-handed states are embedded into the same GUT multiplet. But in the $SU(3)_H$, left-handed states and right-handed states both transform as **3**. The GUT multiplet will contain both **3** and $\bar{\mathbf{3}}$ and the horizontal gauge symmetry model is not consistent with the minimal GUT. Therefore, we will not assume any correlation among rotations of quarks and leptons.

scale, by using the relevant hadronic matrix elements [39]

$$\langle B_q|O_{LL/RR}^{bq}|\bar{B}_q\rangle \approx \frac{1}{3}m_{B_q}f_{B_q}^2 B_{LL/RR}^{bq}$$

$$\langle B_q|O_{LR}^{bq}|\bar{B}_q\rangle \approx -\frac{1}{6}m_{B_q}f_{B_q}^2 B_{LR}^{bq}$$

$$\langle B_q|\widetilde{O}_{LL}^{bq}|\bar{B}_q\rangle \approx \frac{1}{4}m_{B_q}f_{B_q}^2 \widetilde{B}_{LR}^{bq} \tag{22}$$

Here we use $m_{B_q}^2/(m_b+m_q)^2 \approx 1$ and assume $B_{LR}^{bq} \simeq \widetilde{B}_{LR}^{bq} \simeq B_{RR}^{bq} = B_{LL}^{bq} \equiv B_{B_q}$. Then we obtain M_{12}^q

$$M_{12}^q \equiv \langle B_q|\mathcal{H}|\bar{B}_q\rangle = -\frac{1}{3}f_{B_q}^2 m_{B_q} B_{B_q}\left[C_{LL}^{bq}(\mu) + C_{RR}^{bq}(\mu) - \frac{C_{LR}^{bq}(\mu)}{2} + \frac{3\tilde{C}_{LR}^{bq}(\mu)}{4}\right] . \tag{23}$$

From the discussion above, the flavor off diagonal coupling between the horizontal gauge boson Z' and the first two generation quarks are highly suppressed (For the CKM like rotation matrix, it is at least λ^3 suppressed), so we will neglect the new physics contributions to the $K - \bar{K}$ [7] and $D - \bar{D}$ mixing. The Z'-b-s and Z'-b-d couplings, on the other hand, is either unsuppressed or λ suppressed, hence we expect large new physics contributions to modify the magnitudes and the phases of $M_{12}^{d/s}$, where $M_{12}^{d/s}$ are off-diagonal mixing matrix elements in Eq.23. We can parametrize such effects by [8]

$$M_{12}^{d/s} \equiv (M_{12}^{d/s})^{\text{SM}}\Delta_{d/s} \qquad \Delta_s \equiv |\Delta_{d/s}|e^{i\phi_{d,s}^{\Delta}}. \tag{24}$$

The experimental measured observables are summarized as follows, $\Delta m_{d/s}$ and $\Delta\Gamma_{d/s}$ measures the mass and decay width difference between the heavy and light mass eigenstates of the $B_{d/s}$ mesons [9]. $a_{SL}^{d/s}$ is the charge asymmetry in semileptonic $B_{d/s}$ decays. β_d or β_s measure the time-dependent CP violating phases in the hadronic B decay channel $B_d \to$

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[7] The constraint from the CP violation in the $K - \bar{K}$ mixing [28] is, in fact, marginal for $\lambda \simeq 0.1$, $g_H \simeq 0.02$ and $M_{Z'} \simeq 1$ TeV [1].

[8] The CP violation phase is defined as $\phi \equiv \text{Arg}(-M_{12}/\Gamma_{12})$ so we choose Γ_q to be real here.

[9] The experimental uncertainty in the measurements $\Delta\Gamma_d = \text{sign}(\text{Re}\lambda_{CP})\Delta\Gamma_d/\Gamma_d = 0.009 \pm 0.037$. is too big for us to consider it.
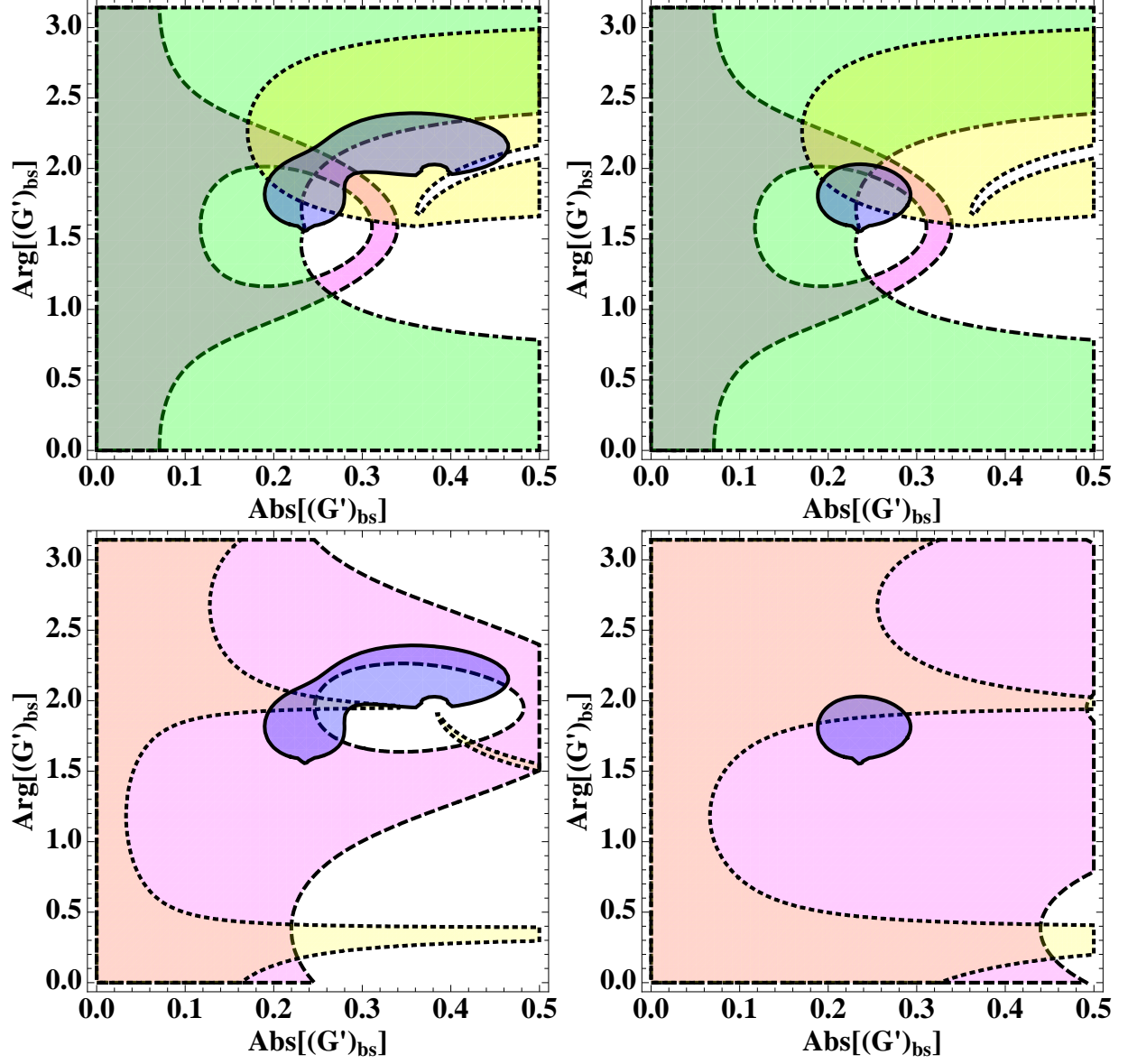
FIG. 1: The $\mathrm{Abs}(G'_{bs})$ versus $\mathrm{Arg}(G'_{bs})$ region plot for $M_{Z'} = 1$ TeV and $g_H = 0.02$. In the left panel, we choose $\mathrm{Abs}(G'_{bd})/\mathrm{Abs}(G'_{bs}) = 0.1$ and $\mathrm{Arg}(G'_{bd}) = \mathrm{Arg}(G'_{bs})$ while in the right panel, we choose $\mathrm{Abs}(G'_{bd})/\mathrm{Abs}(G'_{bs}) = 0.05$ and $\mathrm{Arg}(G'_{bd}) = \mathrm{Arg}(G'_{bs})$. The magenta, green, and yellow region with dashed, dot-dashed and dotted boundary stands for the allowed parameter space at 90% C. L. for $\Delta m_{s/d}$, $\Delta\Gamma_s$ and $S_{\psi K}/\beta_s$.
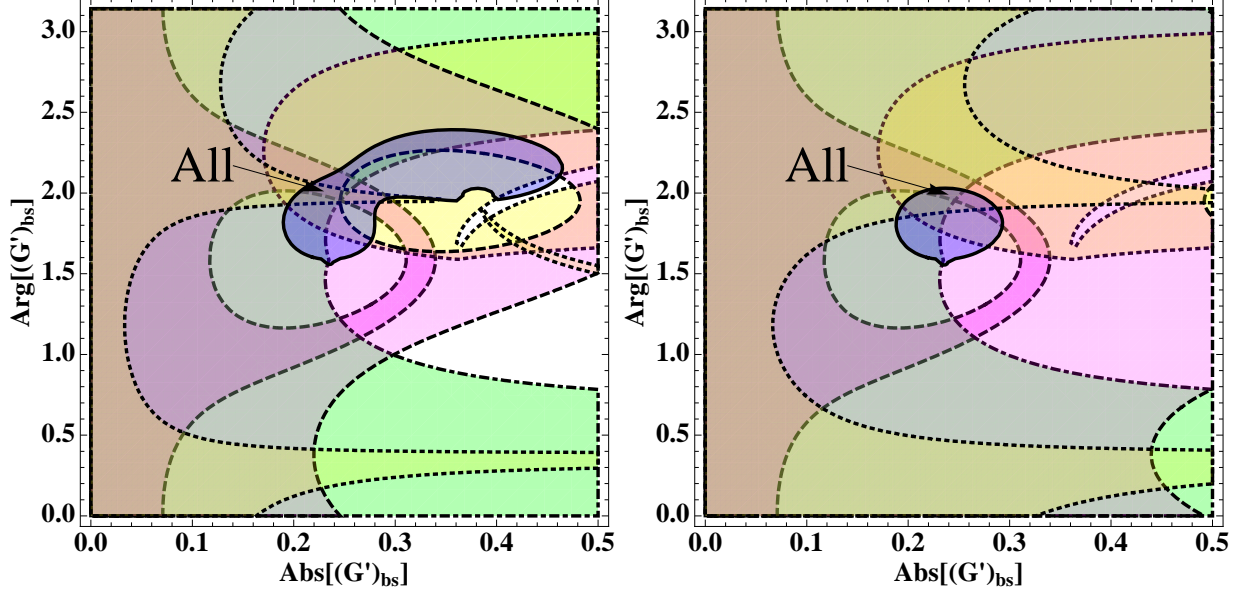
FIG. 2: The similar plots as FIG. 1 but combine both the B_d and B_s experimental constraints. The small overlapped region labeled with "All" is the parameter space that fits for all experimental constraints at 90% C.L.

$J/\psi \, K_S$ or $B_s \to J/\psi \, \phi$. They are shifted by the CPV phases in B_d or B_s mixing.

$$
\begin{aligned}
\Delta m_{d/s} &= \Delta m_{d/s}^{\text{SM}} \left| \Delta_{d/s} \right|, \\
\Delta \Gamma_s &= \Delta \Gamma_s^{\text{SM}} \cos(\phi_s^{\text{SM}} + \phi_s^{\Delta}), \\
a_{\text{SL}}^{d/s} &= \frac{\Delta \Gamma_{d/s}^{\text{SM}}}{\Delta m_{d/s}^{\text{SM}}} \frac{\sin(\phi_{d/s}^{\text{SM}} + \phi_{d/s}^{\Delta})}{|\Delta_{d/s}|}, \\
S_{\psi K} &= \sin\left(2\beta_d + \phi_d^{\Delta}\right), \\
2\beta_s^{\text{Exp}} &= 2\beta_s - \phi_s^{\Delta}.
\end{aligned}
\tag{25}
$$

The theoretical inputs are listed in the Table I. All the decay constants and bag parameters are used from Ref. [37]. Notice that we use the calculations for $\Delta m_{B_s} = 2|M_{12}^s|^{SM}$, $\Delta m_{B_s} = 2|M_{12}^s|^{SM}$ and ϕ^s in Ref. [36] which uses the more recent decay constants and bag parameters with much smaller uncertainties. All of the rest SM inputs are either from [8] or [6]

All experimental measurements which are used to compare with our model outputs are listed in Table II. For the like-sign dimuon charge asymmetry, we use $A_{SL}^b \simeq -(8.5 \pm 2.8) \times 10^{-3}$ which combine the DØ measurements with the CDF measurements. For β_s^{Exp} and Γ_s measured by both CDF and DØ [12–15], we use the combined results with each measurements

TABLE I: The theoretical input parameters [8].

β_d	β_s	m_{B_d}	m_{B_s}
0.38 ± 0.01 [9]	0.018 ± 0.001	5.28 GeV	5.37 GeV
$f_{B_d}\sqrt{\hat{B}_d}$	$f_{B_s}\sqrt{\hat{B}_s}$	f_{B_d}	f_{B_s}
(216 ± 15) MeV	(275 ± 13) MeV	192.8 ± 9.9 MeV	238.8 ± 9.5 MeV
ϕ_d^{SM}	ϕ_s^{SM}	$(\Delta m_{B_d})^{\mathrm{SM}}$	$(\Delta m_{B_s})^{\mathrm{SM}}$
$-0.091^{+0.026}_{-0.038}$	$(4.2 \pm 1.4) \times 10^{-3}$	0.53 ± 0.12 ps^{-1}	19.30 ± 2.2 ps^{-1}
$(\Delta\Gamma_d)^{\mathrm{Exp}}$	$(\Delta\Gamma_s)^{\mathrm{Exp}}$		
$(2.67^{+0.58}_{-0.65}) \times 10^{-3}$ ps^{-1}	0.098 ± 0.024 ps^{-1}		

in 2.8 fb^{-1} [14] where do not add the most recent CDF results [15] since it is difficult for us to extrapolated their contributions between $2.8 - 5.2$ fb^{-1}. Nevertheless, we find that our conclusion in the paper does not change if we use the combined results in Ref. [20] [10].

The experimental constraints on the parameter space of our model are presented in Fig 1 and 2. The parameter $\mathrm{Abs}(G'_{bs})$ and $\mathrm{Arg}(G'_{bs})$ are quite similar as the parameter h_s^2 and σ_s in Ref. [25] except for a overall factor with very small phase related to the $(M_{12}^s)^{\mathrm{SM}}$, therefore our results here can be used as the model independent analysis after some re-parametrization. For illustration, we choose parameter for B_d couplings $\mathrm{Abs}(G'_{bd}) = \mathrm{Abs}(G'_{bs})/10$, $\mathrm{Arg}(G'_{bd}) = \mathrm{Arg}(G'_{bs})$ in which there are sizable contribution to A^b_{sl} from B_d and $\mathrm{Abs}(G'_{bd}) = \mathrm{Abs}(G'_{bs})/20$, $\mathrm{Arg}(G'_{bd}) = \mathrm{Arg}(G'_{bs})$ in which the a^d_{sl} contribution to A^b_{sl} is negligible. In contrast to the paper [18], it is clearly that from the upper right plot in FIG 1, there is no region allowed by all experimental constraints within the 1 σ [11]. The best fitted region for the phase $\mathrm{Arg}(G'_{bs}) \subset (\pi/2, 3\pi/4)$ are quite consistent with the one found in Ref. [25]. However, since the Ref. [25] essentially marginalize over Γ_q^{12} in the range $0 - 0.25$ ps^{-1} and use the best fit points in which $\Delta\Gamma_s$ is about 2.5 times larger than the prediction, the goodness of fit in our

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[10] For the β_s^{Exp} at 90% C. L., we find $\beta_s^{\mathrm{Exp}} \subset (0.13, 0.74)$ from Fig. 7 in Ref. [14] since 1 - CL is almost linear to $\mathrm{Log}(L)$ where L is the likelihood ratio.

[11] We notice that in [18], they use SM prediction directly from [8] which use the old decay constants and bag parameters. Nevertheless, we find that our conclusion still holds because the parameter space mentioned in [18] are not allowed by $(\Delta m_{B_s})^{\mathrm{Exp}}$ and $(\Delta\Gamma_s)^{\mathrm{Exp}}$. In the other paper which contains the model independent fit [19], they simply neglect all the large uncertainties from the SM prediction which lead to the wrong upper bound of A^b_{sl}.

result here is reduced significantly comparing to the one in Ref. [25].

<div align="center">TABLE II: The experimental data.</div>

$S_{J/\Psi K_S}^{\text{Exp}}$	$(\beta_s)^{\text{Exp}}$	$(\Delta m_{B_d})^{\text{Exp}}$	$(\Delta m_{B_s})^{\text{Exp}}$	$(\Delta\Gamma_s)^{\text{Exp}}$
0.655 ± 0.024 [10]	$0.44^{+0.17}_{-0.18}$	0.507 ± 0.005 ps^{-1}	17.77 ± 0.12 ps^{-1}	$0.154^{+0.054}_{-0.07}$ ps^{-1}
A_{sl}^b	$\mathcal{B}^{\text{Exp}}(B_s \to e^+e^-)$	$\mathcal{B}^{\text{Exp}}(B_s \to \mu^+\mu^-)$	$\mathcal{B}^{\text{Exp}}(B_s \to e^\pm\mu^\mp)$	$\mathcal{B}^{\text{Exp}}(B^0 \to \mu^\pm\tau^\mp)$
$-(8.5 \pm 2.8) \times 10^{-3}$ ps^{-1}	$< 5.4 \times 10^{-5}$	$< 4.7 \times 10^{-8}$	$< 6.1 \times 10^{-6}$	$< 3.8 \times 10^{-5}$

B. Meson Decays

The new horizontal gauge boson can also mediate meson hadronic decay or leptonic decays at tree level. In this session, we choose to discuss the two leading processes, $b \to sc\bar{c}$ and $b \to s\mu^\pm\tau^\mp$ respectively to illustrate how meson decays constrain the horizontal gauge interaction. The other transitions are always with additional factors of power of λ suppressions.

The effective $\Delta F = 1$ Hamiltonian responsible for neutral meson decay is

$$\mathcal{H}_{\text{eff}}^{\Delta F=1} = C_3 Q_3 + C_5 Q_5 + \tilde{C}_3 \tilde{Q}_3 + \tilde{C}_5 \tilde{Q}_5 \tag{26}$$

and the leptonic decay like $B_{d,s}^0 \to \ell^+\ell^-$,

$$\mathcal{H}_{\text{eff}}^{\ell} = C_9 Q_9 + C_{10} Q_{10} + \tilde{C}_9 \tilde{Q}_9 + \tilde{C}_{10} \tilde{Q}_{10} \tag{27}$$

where $i, j = e, \mu, \tau$ as lepton flavor indices.

In the case of $b \to sc\bar{c}$ transition, SM contribution at tree level is induced via weak charged current with a CKM factor $V_{bc}V_{sc}^* \sim \lambda^2$. Reading from the effective couplings, the horizontal gauge boson mediated $b \to sc\bar{c}$ has a factor of $G_{bs}G_{cc} \sim \lambda^2$. The SM and horizontal gauge interaction contributions are at the same order of λ and the one can simply compare their couplings and gauge boson masses to estimate the ratio. As we discuss in previous section, the $U(1)_H$ is broken at $\mathcal{O}(\text{TeV})$ which results in a suppression due to $(g_H/g)^4(m_W^4/M_{Z'}^4) \sim 10^{-8}$. Consequently, the contribution to $b \to sc\bar{c}$ from new horizontal gauge boson is completely negligible. In the case of neutral meson mixing, SM leading contribution is from box-diagram while the horizontal gauge boson contribution is at tree

level. Therefore, even if the new horizontal gauge boson is of order TeV, it is still possible to change the ΔM significantly. For decay process, if there exists SM tree level, the above argument then always applies. We won't discuss any constraint from such decays.

Since the horizontal gauge boson also couples to the leptons, there is again tree level contribution to the meson leptonic decay. Within the framework of SM, B_s pure leptonic decays are realized via the electroweak penguin diagrams with $Z/\gamma^* \to \ell^+\ell^-$ and therefore, there is no lepton flavor violation at all. The constraints on leptonic decay are mostly on leptons directly decaying from B meson. If there exists a τ in the final state, τ decay will complicate the search due to the $D^\pm$ decays. Therefore, the leading constraints are

$$\mathrm{Br}(B_s \to \mu^+\mu^-) < 4.7 \times 10^{-8}$$
$$\mathrm{Br}(B_s \to e^+e^-) < 5.4 \times 10^{-5}$$
$$\mathrm{Br}(B_s \to e^\pm\mu^\mp) < 6.1 \times 10^{-6} \tag{28}$$

In our model, the horizontal gauge boson has maximal coupling to b, s and μ, τ and the leading leptonic decay constraint is from $B_s \to \mu^\pm\tau^\mp$. But as we mentioned, $B_s \to \mu^\pm\tau^\mp$ does not exist in SM physics at leading order and it is only from the new physics contribution. Using $\langle 0 \mid \bar{s}\gamma^\mu\gamma_5 b \mid \bar{B}^0 \rangle = \imath f_B p_B^\mu$, one can compute the decay BR as

$$\mathrm{Br}(B_s \to \mu^\pm\tau^\mp) = \tau_B \Gamma(B^0 \to \mu^\pm\tau^\mp) = f_{B_s}^2 \tau_B \frac{m_B^3}{64\pi}\left(1 - \frac{m_\tau^2}{m_B^2}\right)^3 \left(\frac{m_\tau}{m_B}\right)^2 C_{9,10}^{\mu\tau\,2} \tag{29}$$

where the Wilson coefficients $C_{9,10} = g_H^2(m_b)/m_{Z'}^2$. To estimate the decay BR, we take $M_{Z'} \simeq 10^3$ GeV, $g_H \simeq 0.02$ and substitute $m_b = 4.7$ GeV, $m_\tau = 1.777$ GeV, $f_{B_s} = 230$ MeV, $m_B = 5.3$ GeV, $\tau_B = 1.6$ ps, we obtain

$$\mathrm{Br}(B_s \to \mu^\pm\tau^\mp) \simeq 1.2 \times 10^{-9}. \tag{30}$$

[12] The only relevant search is from CLEO as of $B_d \to \mu^\pm\tau^\mp$ [30],

$$\mathrm{Br}(B_d \to \mu^\pm\tau^\mp) < 3.8 \times 10^{-5} . \tag{31}$$

Due to horizontal gauge boson given, the B_d decay partial width has an additional λ^2 suppression so the bound is prediction well below the experimental bound. One can also

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[12] The result is based on assumption from left-handed chiral interaction only which is sufficient to estimate the maximal value of leptonic decay BR.

estimate the $B_s \to \mu^+ \mu^-$ using the above result. The $B_s \to \mu^+ \mu^-$ has a factor of λ^4 suppression then the prediction is about two orders lower than the current experimental bound.

The other possible rare decay which can be induced by the horizontal gauge boson is the FCNC decay in top quark, for instance, $t \to c/u + \mu^\pm \tau^\mp$. However, given the large $M_{Z'}$, the three body decay is highly suppressed.

$$\Gamma(t \to c/u + \mu^\pm \tau^\mp) = \frac{G_F^2 m_t^5}{192\pi^3} \left(\frac{m_W}{M_{Z'}}\right)^4 \left(\frac{g_H}{g}\right)^4 \sim 1.7 \times 10^{-13} \tag{32}$$

Even at the top factory like Large Hadron Collider, it is impossible to observe such rare decay event.

IV. CONCLUSION

The dimuon asymmetry reported by DØ Collaboration is much larger than the SM prediction which suggests new sources for CP violation. In this paper, we propose the possibility to explain such an anomaly through a tree level exchange of a gauged $U(1)_H$ horizontal symmetry in B meson mixing. The $U(1)_H$ horizontal symmetry is a remnant symmetry of $SU(3)_H$ broken at $M_R \sim \mathcal{O}(10^{14})$ GeV through a sextet scalar which gives the neutrino mass. Such a $U(1)_H$ gauge boson only couples to $b - s$ and $\mu - \tau$ in the gauge eigenstate which suppresses all other dangerous meson mixings and B meson decays after the flavor rotation. For a general flavor rotation matrix we consider there is a parameter region around the phase $\text{Arg}(G'_{bs}) \subset (\pi/2, 3\pi/4)$ which fits the data at 90% C. L. For the B decay, the dominate enhanced channel is the $B_s \to \mu^\pm \tau^\mp$. Nevertheless, such a enhanced decay channel is still one order of magnitude smaller than the current experimental bound.

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Appendix A: RG Running of the $\Delta B = 2$ Operators

In the previous section, the Wilson coefficients are given at the M_S scale while to calculate the physics processes involving low energy mesons, one will need to calculate relevant Wilson coefficients at the low energy scale. The running contains two steps, the first step is from $M_{Z'}$ scale to m_t where six flavors contribute to the running of α_s, the second step is from m_t to m_b, where only five flavors contribute. We summarize the running effects of the relevant $\Delta F = 2$ operators below [39].

The operators belonging to the LL/RR, LR sectors read

$$
\begin{aligned}
Q_{LL} &= (\bar{s}^\alpha \gamma_\mu P_L b^\alpha)(\bar{s}^\beta \gamma^\mu P_L b^\beta), \\
Q_{RR} &= (\bar{s}^\alpha \gamma_\mu P_L b^\alpha)(\bar{s}^\beta \gamma^\mu P_L b^\beta), \\
Q_{LR} &= (\bar{s}^\alpha \gamma_\mu P_L b^\alpha)(\bar{s}^\beta \gamma^\mu P_R b^\beta). \\
\tilde{Q}_{LR} &= (\bar{s}^\alpha P_L b^\alpha)(\bar{s}^\beta P_R b^\beta).
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
C_{LL}(\mu_b) &= [\eta(\mu_b)] C_{LL}(\mu_t), \\
C_{RR}(\mu_b) &= [\eta(\mu_b)] C_{RR}(\mu_t), \\
\begin{pmatrix} C_{LR}(\mu_b) \\ \tilde{C}_{LR}(\mu_b) \end{pmatrix} &= \begin{pmatrix} [\eta_{11}(\mu_b)] & [\eta_{12}(\mu_b)] \\ [\eta_{21}(\mu_b)] & [\eta_{22}(\mu_b)] \end{pmatrix} \cdot \begin{pmatrix} C_{LR}(\mu_t) \\ \tilde{C}_{LR}(\mu_t) \end{pmatrix}
\end{aligned}
\tag{34}
$$

The variables η_i is defined as ratio between strong coupling constant at different scales. Given the $U(1)_H$ gauge boson is around 10 TeV, we have two different running of α_s taking into account the threshold correction due to top quark. For the running between the B physics scale and top quark, we have

$$
\eta_5 \equiv \frac{\alpha_s^{(5)}(\mu_t)}{\alpha_s^{(5)}(\mu_b)}
\tag{35}
$$

For the explicit form of η, we list both the expression at the LO (with subscript (0)) and

NLO (with subscript (1))

$$\eta^{(0)}(\mu_b) = \eta_5^{6/23} ,$$

$$\eta^{(1)}(\mu_b) = 1.6273(1 - \eta_5)\eta_5^{6/23} ,$$

$$\eta_{11}^{(0)}(\mu_b) = \eta_5^{3/23} ,$$

$$\eta_{12}^{(0)}(\mu_b) = 0 ,$$

$$\eta_{21}^{(0)}(\mu_b) = \frac{2}{3}(\eta_5^{3/23} - \eta_5^{-24/23}) ,$$

$$\eta_{22}^{(0)}(\mu_b) = \eta_5^{-24/23} ,$$

$$\eta_{11}^{(1)}(\mu_b) = 0.9250\eta_5^{-24/23} + \eta_5^{3/23}(-2.0994 + 1.1744\eta_5) ,$$

$$\eta_{12}^{(1)}(\mu_b) = 1.3875(\eta_5^{26/23} - \eta_5^{-24/23}) ,$$

$$\eta_{21}^{(1)}(\mu_b) = (-11.7329 + 0.7829\eta_5)\eta_5^{3/23} - \eta_5^{-24/23}(-5.3048 + 16.2548\eta_5) ,$$

$$\eta_{22}^{(1)}(\mu_b) = (7.9572 - 8.8822\eta_5)\eta_5^{-24/23} + 0.9250\eta_5^{26/23} . \tag{36}$$

Similarly, for the running between the top quark mass scale and the horizontal gauge boson scale, one can replace all the μ_b, μ_t by μ_t, $\mu_{M_{Z'}}$ in Eq. (34), where all the ηs are

$$\eta^{(0)}(\mu_b) = \eta_6^{6/21} ,$$

$$\eta^{(1)}(\mu_b) = 1.3707(1 - \eta_6)\eta_6^{6/21} ,$$

$$\eta_{11}^{(0)}(\mu_b) = \eta_6^{3/21} ,$$

$$\eta_{12}^{(0)}(\mu_b) = 0 ,$$

$$\eta_{21}^{(0)}(\mu_b) = \frac{2}{3}(\eta_6^{3/21} - \eta_6^{-24/21}) ,$$

$$\eta_{22}^{(0)}(\mu_b) = \eta_6^{-24/21} ,$$

$$\eta_{11}^{(1)}(\mu_b) = 0.9219\eta^{-24/21} + \eta_6^{3/21}(-2.2194 + 1.2975\eta_6) ,$$

$$\eta_{12}^{(1)}(\mu_b) = 1.3828(\eta_6^{24/21} - \eta_6^{-24/21}) ,$$

$$\eta_{21}^{(1)}(\mu_b) = (-10.1463 + 0.8650\eta_6)\eta_6^{3/21} + \eta_6^{-24/21}(-6.4603 + 15.7415\eta_6) ,$$

$$\eta_{22}^{(1)}(\mu_b) = (9.6904 - 10.6122\eta_6)\eta_6^{-24/21} + 0.9219\eta_6^{24/21} , \tag{37}$$

and

$$\eta_6 \equiv \frac{\alpha_s^{(6)}(\mu_h)}{\alpha_s^{(6)}(\mu_t)} . \tag{38}$$

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